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ON

AN INVESTIGATION OF NONLINEAR INTERACTION PHENOMENA IN THE IONOSPHERE

This report covers the period December 1, 1965 to June 1, 1966

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H. C. Hsieh and J. E. Rowe, "Nonlinear Electromagnetic Wave Propagation in a Plasma", Electron Physics Laboratory Tech. Report (in preparation).

H. C. Hsieh, "Dispersion Relations for a Finite Temperature Magnetoactive Plasma", Electron Physics Laboratory Tech. Report (in preparation).

H. C. Hsieh, "Study of the Effects of Electrostatic Fields on the Propagation of Electromagnetic Waves in a Finite Temperature Magnetoactive Plasma", Electron Physics Laboratory Tech. Report (in preparation).

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1. General Introduction (J. E. Rowe)

The overall objective of this research investigation is to study ionospheric phenomena such as thermal radiation noise, the propagation of naturally occurring radio noise through the ionosphere and the generation of VLF emissions. In particular, various traveling-wave interaction mechanisms are being studied in order to determine their applicability in explaining ionospheric phenomena such as those indicated above.

Many of the phenomena of interest involve the interaction between drifting streams of charged particles and electromagnetic waves, including the excitation of electrostatic plasma oscillations and cyclotron-wave interactions. Since many of these phenomena are nonlinear it is reasonable to apply the nonlinear method of analysis used on TWT's and beam-plasma interactions to their study. The two nonlinear methods being used are a Lagrangian particle analysis and an analysis in which the nonlinear collisionless Boltzmann equation is solved by a moment expansion method.

The following sections of this report outline the particular problems presently being investigated. The research activities during the period from December 1, 1965 to June 1, 1966 have been concerned with the study of the propagation of naturally occurring radio noise through the ionosphere.

2. Nonlinear Electromagnetic Wave Propagation in a Finite Temperature Magnetoactive Plasma (H. C. Hsieh and J. E. Rowe)

A properly constructed stationary solution of the nonlinear Boltzmann-Vlasov equation in a moving frame of reference is used to develop a set of ordinary nonlinear differential equations governing the potential functions associated with a one-dimensional electromagnetic wave in a finite temperature magnetoactive two-component plasma. Thus the interaction of an electromagnetic wave with the plasma is described in a moving frame of reference. With a properly imposed boundary condition this set of basic interaction equations can be solved numerically to obtain information on the dynamic electric and magnetic fields. An examination of the basic interaction equations for transverse electromagnetic wave propagation has yielded interesting results for the following special cases:

Case A. No static electric or magnetic field.

Case B. A magnetostatic field in the direction of wave propagation.

Case C. Electrostatic and magnetostatic fields in the direction of propagation.

In Case A where no electrostatic or magnetostatic fields are present and assuming overall electrical neutrality and electron temperature anisotropy, it was observed that the wavelengths of the electromagnetic waves depend upon the wave amplitude. In Case B, where a magnetostatic field is directed along the direction of propagation, the transverse electromagnetic wave appears as a circularly polarized sinusoidal wave in a laboratory frame of reference. Again under the condition of electrical neutrality and assuming a small temperature anisotropy, for a zero mean velocity of the plasma along the direction of wave propagation the basi interaction equations lead to the dispersion relation for Alfvén waves. On the other hand, in Case C, where both electrostatic and magnetostatic fields are in the direction of wave propagation, and the drift velocity of the plasma is nonzero, elliptically polarized waves are observed. Thus the presence of a longitudinal electrostatic field has an interesting effect on the polarization of electromagnetic waves.

Since the solution of the derived set of basic interaction equations should provide desirable information with regard to the electromagnetic field it is proposed to carry out detailed numerical calculations for waves in ionospheric plasmas by imposing a set of realistic boundary conditions. The application of this theory to ionospheric phenomena is to be considered in a future report.

3. Coupling Between Transverse Electromagnetic Waves and Longitudinal Waves in a Finite Temperature Magnetoactive Plasma (H. C. Hsieh)

The dispersion relation for a finite temperature two-component plasma subjected to both electrostatic and magnetostatic fields is derived on the basis of the one-dimensional small-signal assumptions. The derived dispersion relation is given in a form in which various characteristic modes of the system can be readily identified. Moreover the form of the dispersion equation is particularly suitable for a study of the effects of transverse electrostatic and magnetostatic fields upon the coupling between the transverse circularly polarized electromagnetic wave and the longitudinal wave. The investigation of the derived dispersion relation shows that coupling of the longitudinal mode to the transverse mode takes place when transverse electrostatic or magnetostatic fields are present in the system. This dispersion relation is examined in detail for two cases of interest under the assumption of a Maxwellian plasma:

1. Longitudinal propagation in the presence of an applied transverse electrostatic field, and

2. Oblique propagation in the absence of an applied electrostatic field.

Applied transverse electrostatic fields have an interesting effect on the propagation characteristics of transverse circularly polarized electromagnetic waves. For example, in the case of a cold plasma this static electric field causes the cutoff frequency of the wave to shift, while for a finite temperature Maxwellian plasma the electrostatic field causes the longitudinal mode to be coupled to the transverse mode. The details will be discussed in a forthcoming technical report entitled "Study of the Effect of Electrostatic Fields on the Propagation of Electromagnetic Waves in a Magnetoactive Finite Temperature Plasma".

The effect of the transverse magnetostatic field on the propagation characteristic and on the coupling between modes is currently being investigated. The following points will be investigated in the future:

1. Energy conversion between modes, and

2. Coupling of modes in the presence of applied transverse electrostatic or magnetostatic fields.

It is of interest to note that this type of coupling mechanism (electrostatic coupling, or magnetostatic coupling) may be important in ionospheric phenomena. For example, the cutoff, amplification and Landau damping of Whistler propagation in ionospheric plasma may be explained on the basis of this type of coupling mechanism.

4. Application of TWT Theory to the Study of VLF Emissions

(R. J. Lomax)

The treatment by Hsieh and Rowe[1] of the interaction of an electromagnetic wave and a drifting electron stream can be made somewhat more complete by interpreting the dispersion relation kinematically in the manner of Briggs[2] and Sudan[3]. Since only necessary conditions for instabilities were developed it is important to establish sufficiency by either gain calculations or further investigation of the dispersion equation.

In particular it appears that the R_U region of Hsieh and Rowe really corresponds to evanescent waves and therefore it cannot be associated with an amplifying process. On the other hand amplification may take place in their R_L region. There are also indications of the existence of a region of nonconvective instability in the backward-wave interaction which corresponds to localized waves growing in time (until limited by nonlinear effects). These may be observable in the ionosphere by a probe which moves through them.

The above conclusions have been derived on the basis of the amplification criteria of Sturrock[4], but since his results are less general

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1. Hsieh, H. C. and Rowe, J. E., "The Necessary Conditions for Amplification of an Electromagnetic Wave Interacting with a Drifting Electron Stream", Tech. Report No. 88, Grant No. NsG 696, Electron Physics Laboratory, The University of Michigan, Ann Arbor; November. 1965.

2. Briggs, R. J., Electron-Stream Interaction with Plasmas, M. I. T. Press, Cambridge, Mass.; 1964

3. Sudan, R. N., "Classification of Instabilities from Their Dispersion Relations", Phys. Fluids, vol. 8, No. 10, pp. 1899-1904; October, 1965.

4. Sturrock, P. A., "Amplifying and Evanescent Waves, Convective and Nonconvective Instabilities", Plasma Phys., J. E. Drummond, Ed., McGraw-Hill, Inc., New York, pp. 124-142; 1961.

than those of Briggs[2] and Sudan[3], it will be necessary to apply their more rigorous approach. This involves a detailed consideration of the dispersion relation

$$D(\omega, k) = 0$$

considered as a multiple-valued conformal mapping of the complex ω-plane into the complex k-plane, and it will require numerical evaluation of a number of typical cases.